Learning on Graphs under Open-World Assumptions

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General Learning Problems

□ Standard Machine Learning Tasks:



□ Two core ML concepts: representation and generalization

Learning under Closed-World Assumptions



Closed-world assumptions:

Input/output space is shared by train and test data $\mathcal{X}_{te} \subseteq \mathcal{X}_{tr}, \mathcal{Y}_{te} \subseteq \mathcal{Y}_{tr}$ Data distribution stays unchanged from train to test $p_{tr}(x, y) = p_{te}(x, y)$

Towards Open-World Learning



Open-world assumptions: from training to testing

Input/output space goes through expansion $\mathcal{X}_{tr} \subset \mathcal{X}_{te}, \mathcal{Y}_{tr} \subset \mathcal{Y}_{te}$ Data distribution shifts with unknown factors $p_{tr}(x, y) \neq p_{te}(x, y)$

From Closed-World to Open-World Learning



How to learn a desirably effective model under distribution shifts?



The challenging open research problems:



Out-of-Distribution Generalization



Out-of-Distribution Data from Open World





Graph data from multiple domains

Dynamic temporal networks

□ Distribution shifts cause different data distributions $P_{train}(\mathcal{D}) \neq P_{test}(\mathcal{D})$ □ New data from unknown distribution are unseen by training

How to guarantee desired performance on data from new distributions?

Challenges of Graph Data Modeling



each instance is drawed from the same data distribution independently (i.i.d.)



instances have inter-connection and cannot be treated as i.i.d. samples □ Graph notation: A graph G = (A, X), adjacency matrix $A = \{a_{uv} | v, u \in V\}$ node features $X = \{x_v | v \in V\}$, node labels $Y = \{y_v | v \in V\}$ $p(\mathbf{G}, \mathbf{Y} | \mathbf{e}) = p(\mathbf{G} | \mathbf{e}) p(\mathbf{Y} | \mathbf{G}, \mathbf{e})$

where e denotes environment (that affects data generation)

Out-of-distribution generalization on graphs:

learn a classifier $\min_{\substack{f \ e \in \mathcal{E}}} \mathbb{E}_{G \sim p(\mathbf{G}|\mathbf{e}=e)} \left[\frac{1}{|V|} \sum_{v \in V} \mathbb{E}_{y \sim p(\mathbf{y}|\mathbf{G}_{\mathbf{v}}=G_v, \mathbf{e}=e)} \underbrace{[l(f(G_v), y)]}_{\text{loss function for}} \right]$

- A graph G can be divided into pieces of ego-graphs $\{(G_v, y_v)\}_{v \in V}$
- The data generation process: 1) the entire graph is generated via $G \sim p(\mathbf{G}|\mathbf{e})$, 2) each node's label is generated via $y \sim p(\mathbf{y}|\mathbf{G}_{\mathbf{v}} = G_v, \mathbf{e})$

node-level prediction

Causal Invariance Principle

There exists a portion of causal information within input ego-graph for prediction task of each individual node

The "causal" means two-fold properties:1) invariant across environments2) sufficient for prediction



causal features

non-causal features



node features $x_v = [x_v^1, x_v^2]$ causal features predictive model $\hat{y}_v = \frac{1}{|N_v|} \sum_{u \in N_v} \theta_1 x_u^1 + \theta_2 x_u^2$ ideal solutions $[\theta_1, \theta_2] = [1, 0]$ non-causal features

Arjovsky, et al., "Invariant Risk Minimization".

Rojas-Carulla, et al., "Invariant models for causal transfer learning".

Explore-to-Extrapolate Risk Minimization

Initial version: jointly minimize the expectation and variance of risks

 $\min_{\theta} \mathbb{V}_{\mathbf{e}}[L(G^e, Y^e; \theta)] + \beta \mathbb{E}_{\mathbf{e}}[L(G^e, Y^e; \theta)]$

Key issue: environment/domain labels for data are unavailable or ambiguous

□ Final version: adversarial training multiple context generators

 $\begin{array}{l} \operatorname{Risk} & \longrightarrow & \min_{\theta} \operatorname{Var}(\{L(g_{w_{k}^{*}}(G),Y;\theta):1 \leq k \leq K\}) + \frac{\beta}{K} \sum_{k=1}^{K} L(g_{w_{k}^{*}}(G),Y;\theta) \\ & \underset{\mathsf{Exploration}}{\mathsf{Exploration}} & \longrightarrow & \text{s. t. } [w_{1}^{*},\cdots,w_{K}^{*}] = \arg_{w_{1},\cdots,w_{K}} \operatorname{Var}(\{L(g_{w_{k}}(G),Y;\theta):1 \leq k \leq K\}) \\ & \underset{\mathsf{vhere}}{\mathsf{L}(g_{w_{k}}(G),Y;\theta)} = L(G^{k},Y;\theta) = \frac{1}{|V|} \sum_{v \in V} l(f_{\theta}(G_{v}^{k})|y_{v}) \\ & \underset{\mathsf{risk}}{\mathsf{function}} \text{ for data under} \\ & \underset{\mathsf{networks}}{\mathsf{for classification}} \mathsf{for classification} \end{array}$

Experiment on Cross-Graph Transfer



EERM achieves up to 7.0% (resp. 7.2%) impv. on ROC-AUC (resp. accuracy) than ERM

Experiment on Temporal Graph Evoluation



EERM achieves up to 9.6%/10.0% impv using GraphSAGE/GPR-GNN as backbones

Graph-Level Distribution Shifts - Molecules

Key observation: the (bio)chemical properties of a molecule are usually associated with a few privileged molecular substructures



the shared hydroxy (-OH)/ carboxy (-COOH) 🛛 📥 good water solubility

Nianzu Yang, et al., "Learning Substructure Invariance for Out-of-Distribution Molecular Representations", in NeurIPS'22

MoleOOD: Learning Substructure Invariance



□ two-stage training strategy to search for optimal parameters

- 1) optimizing the environment-inference model: $\kappa^*, \tau^* \leftarrow \arg \max_{\kappa \in \mathcal{T}} \mathcal{L}_{elbo}(\tau, \kappa; \mathcal{G}^{train})$
- 2) optimizing the molecule encoder and the predictor: $\theta^* \leftarrow \arg \min \mathcal{L}_{inv}(\theta; \mathcal{G}^{train}, \tau)$

Nianzu Yang, et al., "Learning Substructure Invariance for Out-of-Distribution Molecular Representations", in NeurIPS'22

Distribution Shifts in Sequential Prediction

- Traditional models :



Chenxiao Yang, et al., "Towards out-of-distribution sequential event prediction: A causal treatment", in NeurIPS'22

Causal Intervention for Sequential Prediction

- Proposed interventional models :



Chenxiao Yang, et al., "Towards out-of-distribution sequential event prediction: A causal treatment", in NeurIPS'22

Inherent Generalization of GNNs



Key question: Why GNNs are more powerful than MLP?

PMLP: Propagational MLP
 PMLP=MLP during training
 PMLP=GNN during testing

□ Consistent phenomenons across *sixteen* benchmarks:

- PMLP significantly outperforms MLP
- PMLP performs close to GNN



The superiority of GNNs over MLP comes from better test-time generalization

Chenxiao Yang, et al., "Graph Neural Networks are Inherently Good Generalizers: Insights by Bridging GNNs and MLPs", in ICLR'23

Theoretical Understandings of GNNs



□ By NTK theory we prove:

Compared to MLP, GNNs have better extrapolation ability, i.e., generalizing to OOD data outside training support

Theorem 5. Suppose all node features are normalized, and the cosine similarity of node x_i and the average of its neighbors is deonoted as $\alpha_i \in [0, 1]$. Then, the convergence rate for $f_{pmlp}(x)$ is

$$\left| \frac{\left(f_{pmlp}(\boldsymbol{x}_0 + \Delta t \boldsymbol{v}) - f_{pmlp}(\boldsymbol{x}_0) \right) / \Delta t}{c_{\boldsymbol{v}} \sum_{i \in \mathcal{N}_0 \cup \{0\}} (\tilde{d} \cdot \tilde{d}_i)^{-1}} - 1 \right| = O\left(\frac{1 + (\tilde{d}_{max} - 1)\sqrt{1 - \alpha_{min}^2}}{t} \right).$$
(10)

where $\alpha_{min} = \min{\{\alpha_i\}_{i \in \mathcal{N}_0 \cup \{0\}}} \in [0, 1]$, and $\tilde{d}_{max} \ge 1$ denotes the maximum node degree in the testing node \mathbf{x}_0 's neighbors (including itself).

Chenxiao Yang, et al., "Graph Neural Networks are Inherently Good Generalizers: Insights by Bridging GNNs and MLPs", in ICLR'23

From Closed-World to Open-World Learning



How to learn a desirably effective model under distribution shifts?



The challenging open research problems:

How to train a model that can identify OOD data?

> 00D Detection

Out-of-Distribution Detection

training data



testing data



out-of-distribution (OOD) data

in-distribution (IND) data

OOD Detection:

Train a robust classifier that can <u>identify</u> samples from disparate distributions than (in-distribution) training data

perform well on IND testing data
 identify OOD testing data

Model

OOD Detection for Graph Data

 For a classifier f , our goal is to find a proper decision function that returns the estimation score whether the given input is OOD or not:

$$G(\mathbf{x}, \mathcal{G}_{\mathbf{x}}; f) = \begin{cases} 1, & \mathbf{x} \text{ is an in-distribution instance,} \\ 0, & \mathbf{x} \text{ is an out-of-distribution instance,} \end{cases}$$



GNN-based Node-Level Prediction

Adopt graph neural networks (GNNs) to compute node representations:

$$Z^{(l)} = \sigma \left(D^{-1/2} \tilde{A} D^{-1/2} Z^{(l-1)} W^{(l)} \right), \quad Z^{(l-1)} = [\mathbf{z}_i^{(l-1)}]_{i \in \mathcal{I}}, \quad Z^{(0)} = X$$

• The GNN classifier gives a predictive distribution for node labels:

$$p(y \mid \mathbf{x}, \mathcal{G}_{\mathbf{x}}) = \frac{e^{h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})_{[y]}}}{\sum_{c=1}^{C} e^{h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})_{[c]}}} \quad \text{where } \mathbf{z}_{i}^{(L)} = h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})$$

• If we assume $E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}, y; h_{\theta}) = -h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})_{[y]}$ as an energy function, we have

$$p(y|\mathbf{x}, \mathcal{G}_{\mathbf{x}}) = \frac{e^{-E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}, y)}}{\sum_{y'} e^{-E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}, y')}} = \frac{e^{-E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}, y)}}{e^{-E(\mathbf{x}, \mathcal{G}_{\mathbf{x}})}} \quad \textbf{a Boltzmann distribution}$$
$$E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}; h_{\theta}) = -\log \sum_{c=1}^{C} e^{h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})_{[c]}} \quad \textbf{free energy for OOD detection}$$

Energy Models for OOD Detection

- For a given GNN classifier $h_{ heta}(\mathbf{x},\mathcal{G}_{\mathbf{x}})$, we have the initial energy as

 $\mathbf{E}^{(0)} = [E(\mathbf{x}_i, \mathcal{G}_{\mathbf{x}_i}; h_{\theta})]_{i \in \mathcal{I}} \qquad \text{where } E(\mathbf{x}, \mathcal{G}_{\mathbf{x}}; h_{\theta}) = -\log \sum_{i=1}^{C} e^{h_{\theta}(\mathbf{x}, \mathcal{G}_{\mathbf{x}})_{[c]}}$

• Then we consider propagating the energy values along graph structures

$$\mathbf{E}^{(k)} = \alpha \mathbf{E}^{(k-1)} + (1-\alpha)D^{-1}A\mathbf{E}^{(k-1)}$$
 where $\mathbf{E}^{(k)} = [E_i^{(k)}]_{i \in \mathcal{I}}$

Intuition: connected nodes in the graph tend to be sampled from similar distributions

Proposition 1 (informal)

The energy propagation facilitates *consensus* for the OOD estimation results between the target node and its neighboring nodes.

Loss Functions for Training

• If the training data only contains in-distribution data, use supervised loss:

$$\mathcal{L}_{sup} = \sum_{i \in \mathcal{I}_s} \left(-h_{\theta}(\mathbf{x}_i, \mathcal{G}_{\mathbf{x}_i})_{[y_i]} + \log \sum_{c=1}^C e^{h_{\theta}(\mathbf{x}_i, \mathcal{G}_{\mathbf{x}_i})_{[c]}} \right)$$
 GNN-Safe

• If the training data contains extra OOD data, we additionally consider the regularization loss: $\mathcal{L}_{sup} + \lambda \mathcal{L}_{reg}$

$$\mathcal{L}_{ref} = \frac{1}{|\mathcal{I}_s|} \sum_{i \in \mathcal{I}_s} \left(\operatorname{ReLU}\left(\tilde{E}\left(\mathbf{x}_i, \mathcal{G}_{\mathbf{x}_i}; h_{\theta}\right) - t_{in} \right) \right)^2 + \frac{1}{|\mathcal{I}_o|} \sum_{j \in \mathcal{I}_o} \left(\operatorname{ReLU}\left(t_{out} - \tilde{E}\left(\mathbf{x}_j, \mathcal{G}_{\mathbf{x}_j}; h_{\theta}\right) \right) \right)^2$$

Proposition 2 (informal)

The optimal predicted logits given by \mathcal{L}_{sup} is the same as the counterpart of optimal energy by \mathcal{L}_{reg} .

Main Results on Real-World Datasets

Madal			Twit	cch		Arxiv						
Iviouei	OOD Expo	AUROC	AUPR	FPR	ID ACC	AUROC	AUPR	FPR	ID ACC			
MSP	No	33.59	49.14	97.45	68.72	63.91	75.85	90.59	53.78			
ODIN	No	58.16	72.12	93.96	70.79	55.07	68.85	100.0	51.39			
Mahalanobis	No	55.68	66.42	90.13	70.51	56.92	69.63	94.24	51.59			
Energy	No	51.24	60.81	91.61	70.40	64.20	75.78	90.80	53.36			
GKDE	No	46.48	62.11	95.62	67.44	58.32	72.62	93.84	50.76			
GPN	No	51.73	66.36	95.51	68.09	-	-		-			
GNNSAFE	No	66.82	70.97	76.24	70.40	71.06	80.44	87.01	53.39			
OE	Yes	55.72	70.18	95.07	70.73	69.80	80.15	85.16	52.39			
Energy FT	Yes	84.50	88.04	61.29	70.52	71.56	80.47	80.59	53.26			
GNNSAFE++	Yes	95.36	97.12	33.57	70.18	74.77	83.21	77.43	53.50			

OOD detection results on Twitch and Arxiv

- Metric: AUROC, AUPR, FPR for detection scores of IND-Te and OOD-Te samples
- Twitch (multi-graph dataset): use nodes in different graphs for IND/00D
- Arxiv (a temporal graph dataset): use nodes at different times for IND/00D

Energy Score Visualization



Energy propagation and regularization can both help to enlarge the discrimination gap

Generative Models for Graph OOD Detection

• Define the generative models of node features, graph structures and node labels as two-component mixtures.

 $p_{\theta}(\mathbf{A}|\mathbf{X}, \mathbf{e}) = p_{\theta}(\mathbf{A}|\mathbf{X})^{\mathbf{e}} p_0(\mathbf{A}|\mathbf{X})^{1-\mathbf{e}},$ $p_{\theta}(\mathbf{y}|\mathbf{X}, \mathbf{A}, \mathbf{e}) = p_{\theta}(\mathbf{y}|\mathbf{X}, \mathbf{A})^{\mathbf{e}} p_0(\mathbf{y}|\mathbf{X}, \mathbf{A})^{1-\mathbf{e}}.$

Compute the OOD scores for testing data by Bayesian rule:

$$p_{\theta}(\mathbf{e}|\mathbf{A}, \mathbf{X}) = \frac{p_{\theta}(\mathbf{e}, \mathbf{A}, \mathbf{X})}{\sum_{\mathbf{e}} p_{\theta}(\mathbf{e}, \mathbf{A}, \mathbf{X})} = \frac{p(\mathbf{e})p(\mathbf{X})p_{\theta}(\mathbf{A}|\mathbf{X}, \mathbf{e})}{\sum_{\mathbf{e}} p(\mathbf{e})p(\mathbf{X})p_{\theta}(\mathbf{A}|\mathbf{X}, \mathbf{e})}.$$

Theoretical Justifications:

The model can automatically identify outliers in training data and OOD samples from testing data

Zenan Li et al., "GraphDE: A Generative Framework for Debiased Learning and Out-of-Distribution Detection on Graphs", in NeurIPS'22





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How to learn a desirably effective model under distribution shifts?



OOD Extrapolation

The challenging open research problems:

How to train a model that can generalize to OOD data? How to train a model that can identify OOD data? OOD Detection

How to enable a model to handle new unseen entities?

New Entities from Open World

New users/items in recommender systems

New features collected
 by new released
 platforms for decisions

New developed drugs or combinations for treatment



How to handle unseen entities that are not exposed to model training?

Feature Space Extrapolation

Open-world feature extrapolation:





Key questions: Can we enable neural networks to handle augmented input dimensions without re-training?

Input Data as Graphs

□ The input feature-data matrix can be treated as a bipartite graph

Observed Data Matrix

01

 $o_2 \mid 1$

 $f_1 \,\, f_2 \,\, f_3 \,\, f_4 \,\, f_5$

 $0 \ 1 \ 0 \ 1 \ 0$

 $0 \ 1 \ 0 \ 1 \ 1$

 $1\quad 1\quad 0\quad 0$



Advantage of graph representation: Variable-size for features/instances

Key insight:

Convert inferring embeddings for new features to inductive representation on graphs

Extrapolation with Message Passing



Results on Advertisement Click Prediction

Dataset	Backbone	Model	T1	T2	T3	T4	T5	T6	T7	T 8	Overall
Avazu	NN	Base Pooling FATE	0.666 0.655 0.689	0.680 0.671 0.699	0.691 0.683 0.708	0.694 0.683 0.710	0.699 0.689 0.715	0.703 0.694 0.720	0.705 0.697 0.721	0.705 0.697 0.721	$\begin{array}{c} 0.693 \pm 0.012 \\ 0.684 \pm 0.011 \\ \textbf{0.710} \pm 0.010 \end{array}$
	DeepFM	Base Pooling FATE	0.675 0.666 0.692	0.684 0.676 0.702	0.694 0.685 0.711	0.697 0.685 0.714	0.699 0.688 0.718	0.706 0.693 0.722	0.708 0.694 0.724	0.706 0.694 0.724	$\begin{array}{c} 0.697 \pm 0.009 \\ 0.685 \pm 0.009 \\ \textbf{0.713} \pm 0.010 \end{array}$
Criteo	NN	Base Pooling FATE	0.761 0.761 0.770	0.761 0.762 0.769	0.763 0.764 0.771	0.763 0.763 0.772	0.765 0.766 0.773	0.766 0.767 0.774	0.766 0.768 0.774	0.766 0.768 0.774	$\begin{array}{c} 0.764 \pm 0.002 \\ 0.765 \pm 0.001 \\ \textbf{0.772} \pm 0.001 \end{array}$
	DeepFM	Base Pooling FATE	0.772 0.772 0.781	0.771 0.772 0.780	0.772 0.773 0.782	0.772 0.774 0.782	0.774 0.776 0.784	0.774 0.776 0.784	0.774 0.776 0.784	0.774 0.776 0.784	$\begin{array}{c} 0.773 \pm 0.001 \\ 0.774 \pm 0.002 \\ \textbf{0.783} \pm 0.001 \end{array}$

Table. ROC-AUC results for eight test sets (T1 - T8) on Avazu and Criteo

FATE achieves significantly improvements over Base/Pooling with different backbones (DNN and DeepFM)

Input Space Expansion - Cold-Start Users

open-world recommendation: new unseen users appear in test data



□ Challenges: For new users, there is no available embeddings from model training

Can we enable a recommendation model to directly generalize to new users ?

Extrapolation with Graph Structure Learning

□ Basic idea:

- leverage one group of users to express another
- learn a latent graph over users
- message passing from existing users to new ones



Qitian Wu et al., "Towards Open-World Recommendation: An Inductive Model-based Collaborative Filtering Approach", in ICML'21

Key insight: user preferences share underlying proximity that induces latent graphs

Results on Recommendation Benchmarks

□ Task 1: Transferring to few-shot users with limited interaction records

□ Task 2: Generalizing to zero-shot users unseen by training

				Dou	ıban			ML-	100K			ML	-1 M									
Method	Inductive	Feature	RM	ISE	ND	CG	RM	ISE	ND	CG	RM	ISE	ND	CG							0.5	
			All	FS	All	FS	All	FS	All	FS	All	FS	All	FS	0			n trans	0.00050	0.8	³⁵	•
PMF	No	No	0.737	0.718	0.939	0.954	0.932	1.003	0.858	0.843	0.851	0.946	0.919	0.940	× 200				- 0.00045	^{8.0} ۲	- 06	
NNMF	No	No	0.729	0.705	0.939	0.952	0.925	0.987	0.895	0.878	0.848	0.940	0.920	0.937	nde				- 0.00040	eigh		
GCMC	No	No	0.731	0.706	0.938	0.956	0.911	0.989	0.900	0.886	0.837	0.947	0.923	0.939	1900 ISer i				- 0.00035	A 0.7 UO	/5 -	
NIMC	Yes	Yes	0.732	0.745	0.928	0.931	1.015	1.065	0.832	0.824	0.873	0.995	0.889	0.904	n VI5 600				- 0.00030	enti	70 -	and the second
BOMIC	Yes	Yes	0.735	0.747	0.923	0.925	0.931	1.001	0.828	0.815	0.847	0.953	0.905	0.924	one				0.00050	Att		• • • • • • • •
F-EAE	Yes	No	0.738	-	-	-	0.920	-	-	-	0.860	-	-	-	800		181 		- 0.00025	0.6	55 -	
IGMC	Yes	No	0.721	0.728	-	-	0.905	0.997	- 1	-	0.857	0.956	-	-				. it is a	0.00020		Ļ	<u> </u>
IDCF-NN (ours)	Yes	No	0.738	0.712	0.939	0.956	0.931	0.996	0.896	0.880	0.844	0.952	0.922	0.940	1	0 1000 200	00 3000	4000 500	0		0	200 400 600 800 1000 1200 1400 # History ratings
IDCF-GC (ours)	Yes	No	0.733	0.712	<u>0.940</u>	0.956	0.905	<u>0.981</u>	<u>0.901</u>	0.884	<u>0.839</u>	0.944	<u>0.924</u>	<u>0.940</u>		Reyt	iser inc					, 3

+4.0% (resp. +17.4%) impv. of RMSE (resp. NDCG) on new users

Qitian Wu et al., "Towards Open-World Recommendation: An Inductive Model-based Collaborative Filtering Approach", in ICML'21

References

Out-of-Distribution Generalization:

[1] Qitian Wu, et al., Handling Distribution Shifts on Graphs: An Invariance Perspective, in ICLR'22

[2] Nianzu Yang, et al., Learning Substructure Invariance for Out-of-Distribution Molecular Representations, in NeurIPS'22

[3] Chenxiao Yang et al., Towards out-of-distribution sequential event prediction: A causal treatment, in NeurIPS'22

[4] Chenxiao Yang et al., Graph Neural Networks are Inherently Good Generalizers: Insights by Bridging GNNs and MLPs, in ICLR'23

Out-of-Distribution Detection:

[5] Qitian Wu et al., Energy-based Out-of-Distribution Detection for Graph Neural Networks, in ICLR'23
 [6] Zenan Li et al., GraphDE: A Generative Framework for Debiased Learning and Out-of-Distribution Detection on Graphs, in NeurIPS'22

Out-of-Distribution Extrapolation:

[7] Qitian Wu et al., Towards Open-World Feature Extrapolation: An Inductive Graph Learning Approach, in ICML'21

[8] Qitian Wu et al., Towards Open-World Recommendation: An Inductive Model-based Collaborative Filtering Approach, in NeurIPS'21

